Sr.No.3160

Exam Code: 103204
Subject Code : 1106

B.A./B.Sc. - 4th Semester

(2721)

Paper : Mathematics Paper-I (Statics and Vector Calculus)

Time allowed: 2 hrs. **Max. Marks: 50**

Note: There are EIGHT questions of equal marks. Candidates are required to attempt any FOUR questions.

Section A

1.a). The magnitude of the resultant of two forces$\vec{P}$ and $\vec{Q}$ is $(2m+1)\sqrt{P^2+Q^2}$ when acting at an angle θ and $(2m-1)\sqrt{P^2+Q^2}$ when acting angle $\frac{\pi}{2}-\theta$. Show that$tan\theta=\frac{m-1}{m+1}$.

(b) State and prove Lami's theorem.

2 (a)The forces of 1 ,2,3,5,P,Q act along AB, BC ,CD, DA, AC and BD resp. and ABCD is a square of side a. Find the value of P and Q for the system to reduce to a couple. Find also the moment of the couple.

(b) A uniform beam of length 2a rests against a smooth vertical plane over smooth peg at a distance b from the plane. If θ be the Inclinations of beam to the vertical, show that

$$sini^3\,\theta=\frac{b}{a}$$

Section B

3 (a) If the force which acting parallel to a rough plane of inclinations α to the horizon is just sufficient to draw a weight up be n times the force which will just be on the point of sliding down show that

$Tan\alpha=\frac{\mu(n+1)}{n-1}$ where μ is the coefficient of friction.

(b) A uniform ladder rest an angle $\frac{\pi}{4}$ with the horizontal with its upper extremity against a rough wall and its lower extremity on the rough ground with coefficients of frictions $\mu and\ \mu'$ respectively. Show that the least horizontal force which would move the lower extremity toward the wall is

$\frac{\frac{1}{2}W(1+2\mu-\mu\mu')}{1-\mu'}$, W is weight.

4 (a)A uniform wire is bent in the form of triangle with sides a, b, c. Prove that the distances of the centre of gravity of the whole from the sides are in ratio $\frac{b+c}{a}:\frac{c+a}{b}:\frac{a+b}{c}$.

(b) A solid of uniform density is build up of a hemisphere of radius r and a circular cylinder of radius r and height h on the circular base of the hemisphere. Find the position of centre of gravity of solid from the common base.

Contd....P/2

Section C

5 (a)If $\vec{a}$is a constant vector, then show that $\nabla \times \left(\frac{\vec{a}\times\vec{r}}{r^3}\right) = \frac{\vec{a}}{r^3} + \frac{3(\vec{a}.\vec{r})}{r^5}\vec{r}$

(b)$If\ \vec{F} = y\hat{\imath} + (x - 2xz)\hat{\jmath} - xy\hat{k}$ evaluate $\int_S(\nabla \times \vec{F}).\hat{n}ds$ where S is a surface of sphere $x^2 + y^2 + z^2 = a^2$ above XY plane.

6 (a)$If\ \vec{F} = (2x^2 + y^2)\hat{\imath} + (3y - 4x)\hat{\jmath}$ evaluate $\int_C \vec{F}.d\vec{r}$ around the triangle ABC whose vertices are A(0, 0), B(2. 0) and C(2, 1).

(b)Evaluate $\iint_S \vec{F}.\hat{n}ds$ over the surface S at the region bounded by the cylinder $x^2 + z^2 = 9, x = 0, y = 0, z = 0\ and\ y = 8\ where\ \vec{F} = 6z\hat{\imath} + (2x + y)\hat{\jmath} - x\hat{k}$

Section D

7 State and prove Green's theorem in plane.

8 (a)Verify Stokes' theorem for $\vec{F} = (2x - y)\hat{\imath} - yz^2\hat{\jmath} - y^2z\hat{k}\ where\ S\ is\ the\ upper\ half$ surface of sphere $x^2 + y^2 + z^2 = 1$ and C is boundary.

(b)Using Gauss's divergence theorem , evaluate

$\iint_S(ax^2 + by^2 + cz^2)dS$ over the sphere $x^2 + y^2 + z^2 = 1$

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